

Ratios

When you are working with ratio problems, you must always add the individual parts to find the whole.

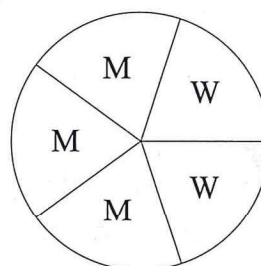
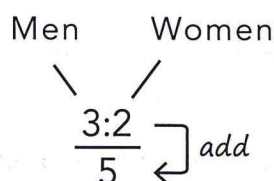
E1 In a certain city the ratio of the number of men to women is 3 to 2. What fraction of the population is female?

- A. $\frac{2}{5}$
- B. $\frac{3}{5}$
- C. $\frac{2}{3}$
- D. $\frac{5}{3}$
- E. $\frac{3}{2}$

S 1 Add the parts to find the whole: $3 + 2 = \boxed{5}$

fraction female = ?

2 Place the ratio of the parts over the whole:



3 Circle your fraction:

Men

$$\frac{3}{5}$$

Women

$$\frac{2}{5}$$

The fraction of the population that is male is $\frac{3}{5}$, and the fraction of the population that is female is $\frac{2}{5}$.



When you're working with ratios, think of a **pie**. Find the total number of pieces in the pie, and you're good to go.

- E2** If a \$345,000 prize was divided among three game show contestants in the ratio 2 to 7 to 9, what was the largest amount awarded to any one contestant, rounded to the nearest dollar?

- F. \$7,041
- G. \$38,333
- H. \$134,167
- J. \$172,500
- K. \$345,000

S

Smallest Largest

$$\frac{2:7:9}{18} \quad \leftarrow \text{add}$$

largest prize = ?

Smallest

$$\frac{2:7:9}{18}$$

Middle

$$\frac{2:7:9}{18}$$

Largest

$$\frac{2:7:9}{18}$$

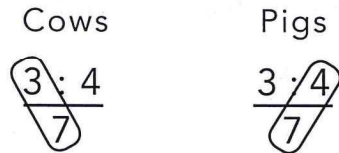
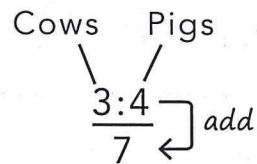
The largest prize awarded is $\frac{9}{18}$, or half of the total prize.

$\frac{1}{2}$ of \$345,000 = **\$172,500**, choice J.

E3

- If a farmer keeps only cows and pigs in the ratio of 3 to 4, each of the following could be the number of animals on the farm EXCEPT

- A. 7
- B. 10
- C. 28
- D. 42
- E. 84

S

Unless we want to have fractions of animals, we need our grand total to be a **multiple of 7**. So cold-cuts aside, we need to see totals such as 7, 14, 21, 28, etc.

For example, if the farmer had 28 animals, $\frac{3}{7}$ would be cows and $\frac{4}{7}$ would be pigs.

$$\text{Cows} \longrightarrow \frac{3}{7} \times 28 = 12 \text{ cows.}$$

$$\text{Pigs} \longrightarrow \frac{4}{7} \times 28 = 16 \text{ pigs.}$$

All whole animals. All good!

So the only odd-ball answer in our current problem is **B**. If we have 10 animals, we get animal parts:

$$\text{Cows} \longrightarrow \frac{3}{7} \times 10 = 4.29 \text{ cows}$$

$$\text{Pigs} \longrightarrow \frac{4}{7} \times 10 = 5.71 \text{ pigs}$$

We want our animals whole! Our answer is **(B)**.

Ratios Practice Problems

1. A catering order calls for Greek and Caesar salads in the ratio of 4:5. If there are 54 salads in the order, how many are Caesar salads?

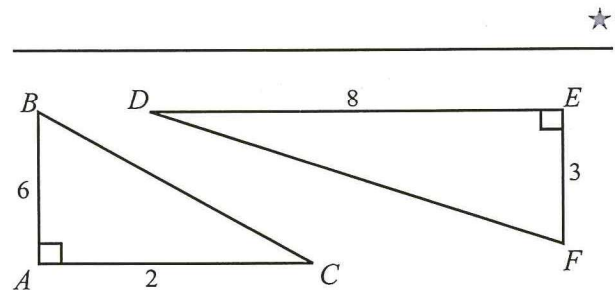
A. 4
B. 5
C. 24
D. 30
E. 54

2. Which of the following is equal to the ratio 3.5 to 14?

F. 1 to 2
G. 1 to 3
H. 8 to 2
J. 2 to 8
K. 2 to 80

3. In the wild, the ratio of male peacocks to female peacocks is 3 to 5. What percent of peacocks in the wild are female?

A. 32.5%
B. 35.0%
C. 37.5%
D. 60.0%
E. 62.5%



4. In the figure above, what is the ratio of the area of triangle ABC to the area of triangle DEF ?

F. 1:2
G. 1:3
H. 1:4
J. 2:3
K. 2:5

5. If Kara only owns drama and comedy movies and the ratio of dramas to comedies in her DVD collection is 3:2, which of the following could be the total number of DVDs in her collection?

A. 15
B. 16
C. 17
D. 18
E. 19

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6. When 30 students were asked who they would vote for in an upcoming class election, their results were put into the table shown below:

Candidate	Number of Voters
Sam	7
Ruth	12
Jordan	11

If the student responses are indicative of how the 500 students in the class will vote in the actual student election, which of the following best estimates the votes that Ruth will receive in the election?

F. 100
G. 150
H. 200
J. 250
K. 300

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7. In a parking lot with 168 spaces, there were 48 more filled spaces than empty spaces. What was the ratio of filled spaces to empty spaces?

A. 5 to 9
B. 9 to 5
C. 2 to 1
D. 5 to 2
E. 11 to 3

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8. The ratio of x to y is 1 to 3, and the ratio of y to z is 2 to 3. What is the ratio of x to z ?

F. 1 to 2
G. 1 to 3
H. 2 to 3
J. 1 to 9
K. 2 to 9

★★★

2%

of all math
questions on
the ACT

Probability

Probability represents the likelihood that a particular event will occur. It is expressed as a value from 0 to 1.

A probability of **1** tells you: *It's a sure thing!*

A probability of **0** tells you: *It's just not going to happen.*

The formula is simple:

$$\text{Probability} = \frac{\text{\# of outcomes that meet the requirements}}{\text{Total \# of possible outcomes}}$$

E1 A shelf contains only plastic toys and wooden toys. If 6 out of 15 toys on the shelf are plastic, what is the probability that a toy selected at random will be wooden?

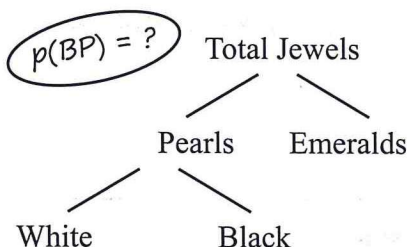
S So 6 out of our 15 toys are plastic; the remaining 9 toys must be wooden. To find the probability that a toy chosen at random will be wooden, we use this equation:

p(wooden) = ?

$$\frac{\text{\# that meet requirements}}{\text{\# of total possible}} = \frac{\text{Wooden Toys}}{\text{Total Toys}} = \left(\frac{9}{15} \right)$$

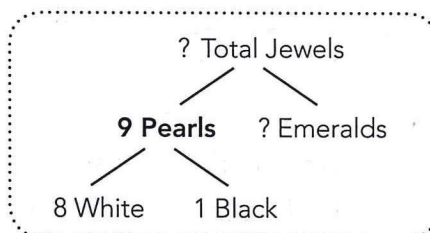
- E2** A bag has only pearls and emeralds inside. There are three times as many pearls as emeralds. The pearls are either white or black, and 8 times as many pearls are white as are black. If one jewel is drawn at random from the bag, what is the probability that the jewel drawn will be a black pearl?

S We can solve this problem by using a probability tree. Build the tree **starting at the top** with the most generic detail (total) down to the most specific detail (White or Black).

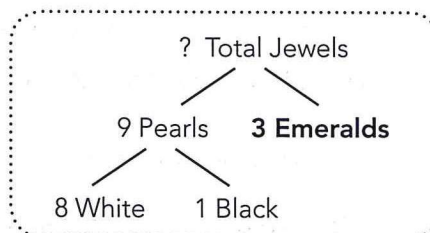


Now let's fill in the information, **starting from the bottom**:

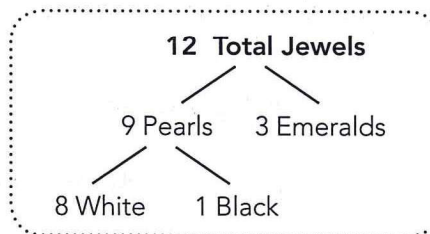
- 1** Starting at the lowest level, we have 8 white pearls for each black pearl. This gives us **9 pearls total**.



- 2** Moving up the tree, we have 3 pearls for every emerald. Since we have 9 pearls, we must have **3 emeralds**.



- 3** Moving up the tree, we know we have 9 pearls and 3 emeralds. That makes a **total of 12 jewels**.



Looking at our tree, it's easy to see that out of 12 jewels, the probability of drawing 1 black pearl is $\frac{\text{Black Pearl}}{\text{Total Jewels}} = \frac{1}{12}$.

Changing Probabilities

Whenever you have a probability problem where one component is changing while the other stays the same, **focus on the element that is not changing**.

E3 There are 90 fish in a pond. 40 are goldfish and 50 are clownfish. If clownfish are added to the pond until the probability of randomly catching a clownfish becomes $\frac{2}{3}$, what will be the total number of clownfish in the pond?

S Let's set up our problem.
In the beginning we have:

clown = ?

<u>Goldfish</u>	<u>Clownfish</u>	<u>Total (T)</u>
40	50	90

Currently the probability of randomly catching a goldfish is $\frac{40}{90}$.
and the probability of randomly catching a clownfish is $\frac{50}{90}$.

We are going to be adding clownfish, but the number of goldfish will stay the same. So let's focus on the **goldfish**.

We will add only clownfish until we have $\frac{2}{3}$ clownfish in the pond.
That means our **original 40 goldfish** will now account for $\frac{1}{3}$ of the total fish in the pond. Let's write that out and solve:

$$40 \text{ goldfish} = \frac{1}{3} \text{ of the new total.}$$

$$40 = \frac{1}{3} \times T$$

$$120 = T$$

$$120 \text{ total fish} - 40 \text{ goldfish} = \text{80 clownfish}$$

- E4** An artist is planning to hold a private art show. Of the pieces he has already completed, 20 are sculptures and 40 are oil paintings. If the artist does not complete any more oil paintings, how many more sculptures must he complete in order to ensure that the probability of seeing a sculpture at the show is $\frac{4}{5}$?

S In the beginning we have:

additional sculptures = ?

<u>Sculptures</u>	<u>Paintings</u>	<u>Total (T)</u>
20	40	60

Currently $\frac{20}{60}$ (or $\frac{1}{3}$) of the pieces are sculptures.

We are going to be adding only sculptures until $\frac{4}{5}$ of the total pieces are sculptures and $\frac{1}{5}$ are paintings.

Let's focus on the **paintings**, since they are the unchanging factor.

When all the works are completed, 40 paintings will comprise $\frac{1}{5}$ of the total pieces in the show.

$$40 = \frac{1}{5} \times T$$

$$200 = T$$

$$200 \text{ total pieces} - 40 \text{ paintings} = 160 \text{ sculptures}$$

We're not done yet! The question asks how many **more** sculptures must he create. Starting from a base of 20, he will need to knock out:

$$160 - 20 = \text{140 additional sculptures}$$

Probability Practice Problems

1. In a certain store, there are 3 blue sweaters, 4 red sweaters, 7 green sweaters, and 4 white sweaters for sale. If these are the only sweaters in the store, what is the probability that a sweater randomly selected from this store is blue?

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{11}{18}$
E. $\frac{2}{3}$

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2. Every student in Mr. Smith's class plays exactly one sport. The probability that a student chosen at random from the class plays baseball is $\frac{1}{8}$. If exactly 3 students play baseball, what is the total number of students in the class?

F. 6
G. 12
H. 18
J. 24
K. 30

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3. A drawer contains only white, black and brown socks. If the probability of randomly selecting a white sock from the drawer is $\frac{1}{5}$ and the probability of selecting a black sock is $\frac{1}{4}$, which of the following could **NOT** be the total number of socks in the drawer?

A. 20
B. 25
C. 40
D. 60
E. 80

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4. The Atlanta Zoo has 3 monkeys that are 10 years old, 6 monkeys that are 6 years old, 2 monkeys that are 4 years old, and 4 monkeys that are 2 years old. What is the probability that a monkey selected at random will be less than 6 years old?

F. $\frac{2}{5}$
G. $\frac{4}{15}$
H. $\frac{3}{5}$
J. $\frac{11}{15}$
K. $\frac{4}{5}$

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Number of Computers per Home in Johnstown

Computers per Home	Number of Homes
0	4
1	16
2	20
3	12
4	8

5. A survey revealed the data shown in the table above. What is the probability that a home chosen at random from Johnstown will contain exactly 3 computers?

- A. $\frac{1}{5}$
 B. $\frac{1}{4}$
 C. $\frac{1}{3}$
 D. $\frac{1}{2}$
 E. $\frac{2}{3}$

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6. If two of the positive factors of 6 are multiplied together, what is the probability that the result will be a multiple of 2?

- F. $\frac{1}{6}$
 G. $\frac{1}{3}$
 H. $\frac{1}{2}$
 J. $\frac{2}{3}$
 K. $\frac{5}{6}$

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	Male	Female
Age 0 – 20	35	93
Age 21 – 40	55	49
Age 41 – 60	60	48

7. The table above shows demographic information for 340 customers at City Mall. What is the probability that a randomly selected person at this mall is a male whose age is 21 or greater?

- A. $\frac{39}{289}$
 B. $\frac{55}{340}$
 C. $\frac{23}{68}$
 D. $\frac{15}{34}$
 E. $\frac{127}{170}$

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8. A closet contains 8 blue shirts, 9 white shirts, and 7 striped shirts. How many additional blue shirts must be added to the 24 shirts in the closet so that the probability of selecting a blue shirt at random is $\frac{3}{7}$?

- F. 2
 G. 4
 H. 6
 J. 8
 K. 10

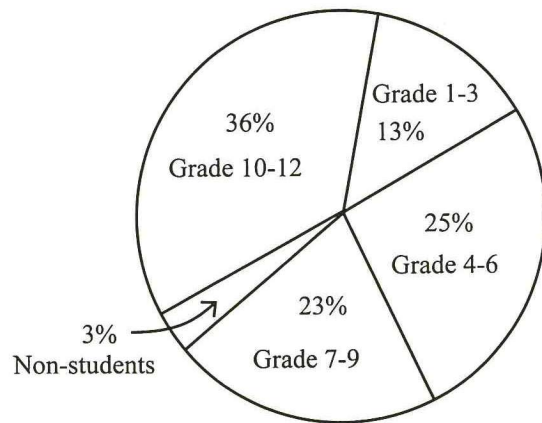
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9. An integer from 500 through 1000, inclusive, will be selected at random. What is the probability that the number selected will have 1 as at least 1 digit?

A. $\frac{11}{501}$
B. $\frac{36}{501}$
C. $\frac{96}{501}$
D. $\frac{101}{501}$
E. $\frac{201}{1000}$

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10. A school held a raffle contest to raise money for a local soup kitchen. The circle graph below shows the distribution of raffle tickets that students purchased according to their grade. A raffle ticket holder is selected randomly from this group to win a prize. What are the odds (in the grade range: not in the grade range) that the person with the winning ticket is in grade 10, 11, or 12?



F. 1 : 3
G. 9 : 16
H. 9 : 37
J. 18 : 33
K. 36 : 25

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